Vibrations of Thin Conical Shells Subjected to Sudden Heating

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The present problem is a study of the response of truncated, thin conical shells subjected to rapid surface heating. The time-dependent temperature gradient across the wall will produce a moment that therefore is included in the equation of motion. The axisymmetric deflection function is divided into two parts in order to satisfy the nonhomogeneous boundary conditions. The Galerkin method is used to integrate the equation of motion, and a second-order differential equation is solved thereafter. The quasi-static solution is a limiting case when the mass density approaches zero. In the numerical example, the conical shell is heated suddenly at one surface and insulated at the other. The natural frequencies and deflections vs various geometr'c parameters are evaluated. For very thin shells, the ratio between the maximum deflection and the maximum static-deflection is nearly two to one.

Nomenclature

A = thermal diffusivity
B = parameter defined in Eq. (39)
E = Young's modulus
M_T = nondimensional thermal moment, Eq. (10)
R = radius at the smaller end
$\bar{T}$ = temperature gradient
h = thickness of shell
m, N = longitudinal mode number = odd integers
n = number defined in Eq. (23)
r, θ = surface coordinates
$\bar{t}$ = dimensional time
t = nondimensional time = $\bar{t}$/unit of time
$\bar{w}, \bar{u}, \bar{v},$ = dimensional displacement components of the middle surface (Fig. 1)
w = $\bar{w}/h$
χ, r_1 = length index and meridional distance, respectively defined in Eq. (4) and Fig. 1
χ_0 = value of χ at the larger end
α = thermal expansion coefficient
β = semivertex angle
ν = Poisson's ratio
$\bar{\omega}$ = natural frequency, rad/sec
ω = nondimensional frequency = $\bar{\omega}\bar{t}/t$
Ω = frequency parameter = $\bar{\omega}h(R/h)^2(\bar{\rho}/E)^{1/2}$
$\bar{\rho}$ = mass density
ρ = nondimensional mass density defined in Eq. (9)

Introduction

THE free vibrations of conical shells have been studied by several investigators in recent years. In Ref. 1, Federhofer determined the natural frequency of conical shell by the use of power series and the Rayleigh-Ritz method. Herrmann and Mirsky[2] studied the free vibrations with consideration of small vertex angle. They assumed sinusoidal mode shapes in the determination of frequencies. Methods of approach for the free vibration problems have been discussed by Shulman.[3] The axisymmetric modes and frequencies of conical shells have been determined by Goldberg, Bogdanoff, and Marcus[4] after numerically integrating a set of first-order differential equations. The same technique has been applied also to study the modes and frequencies of pressurized conical shells.[5] In Ref. 6, Saunders, Wisniewski, and Paslay considered the radial displacement mode shape in polynomial form for the fixed-end conical shells. Garnet and Kempner[7] have studied the axisymmetric free vibrations of conical shells with the influence of the transverse-shear deformation and rotatory inertia. A clamped-end boundary condition was used by Holmes[8] in the case of axisymmetric vibrations of a conical shell supporting a mass. Experimental investigation of vibrational characteristics of cylindrical and conical shells has been made recently by Watkins and Clary[9]. All the aforementioned works are interested in the nature of free vibrations of conical shells.

When a structural element is subjected to a nonuniform temperature field, the thermal effects are not negligible in many cases. If the temperature function varies slowly with time, the quasi-static solution is sometimes satisfactory. But in the case of sudden heating, the dynamic effects caused by the influence of inertia have to be considered. Boley[10] first studied the vibrations of beams that have a transient temperature gradient in the transverse direction. In a later paper,[11] the thermally induced vibrations of rectangular plates were studied by Boley and Barber.

The purpose of the present study is to find the approximate dynamic response of truncated conical shells, one surface of which is subjected to sudden heating. The temperature gradient across the wall thickness will produce a moment that is time-dependent. The symmetric mode vibration is considered. The Galerkin method has been used to integrate the equation of motion. The dynamic deflection response to the heat input is obtained by solving the second-order differential equation. The natural frequency of the free vibration is found from the homogeneous portion of the second-order equation. The quasi-static solution is a limiting case when the mass density approaches zero. Numerical calculations have been made with various geometric parameters of the conical shells. For the purpose of comparison, the temperature field used in the calculation has the same expression as that assumed in Ref. 10. The dynamic effects are apparent from the illustrated results. In order to write the solutions in brief forms, some functions are denoted by symbols. Their expressions are listed in the Appendix.

Basic Equations

The coordinates r, θ and displacement components $\bar{u}$, $\bar{v}$, $\bar{w}$, are defined in the Nomenclature and Fig. 1. The semivertex angle is denoted by β. When the deformation is small, the strain-displacement and strain-stress relations

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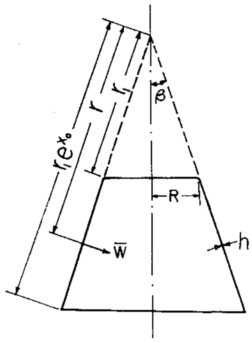

Fig. 1 Geometry of conical shell.

with temperature gradient $\bar{T}$ are

$$
\left.\begin{aligned}
\epsilon_r &= (\partial\bar{u}/\partial r) = (1/E)(\sigma_r - \nu\sigma_\theta) + \alpha\bar{T} \\
\epsilon_\theta &= (\bar{u}/r) - (\bar{w}/r)\cot\beta + (1/r\sin\beta)(\partial\bar{v}/\partial\theta) \\
&= (1/E)(\sigma_\theta - \nu\sigma_r) + \alpha\bar{T} \\
\epsilon_{r\theta} &= (\partial\bar{v}/\partial r)\,(\bar{v}/r) + (1/r\sin\beta)(\partial\bar{u}/\partial\theta) \\
&= [2(1+\nu)/E]\sigma_{r\theta}
\end{aligned}\right\} \tag{1}
$$

where E, α, and ν represent Young's modulus, thermal expansion coefficient, and Poisson's ratio, respectively.

If the longitudinal inertia terms are neglected, the equilibrium conditions in the middle plane can be satisfied by using the Airy stress function.[12] This function is defined as

$$
\left.\begin{aligned}
\sigma_r &= (1/r)(\partial\bar{F}/\partial r) - (1/r^2\sin^2\beta)(\partial^2\bar{F}/\partial\theta^2) \\
\sigma_\theta &= \partial^2\bar{F}/\partial r^2 \\
\sigma_{r\theta} &= -(\partial/\partial r)[(1/r\sin\beta)(\partial\bar{F}/\partial\theta)]
\end{aligned}\right\} \tag{2}
$$

The compatibility equation is established by the following equality:

$$
\left.\begin{aligned}
\frac{r^2}{E}\bar{\nabla}^4\bar{F} &= -\frac{\partial}{\partial r}\left(\frac{r}{\sin\beta}\frac{\partial\epsilon_{r\theta}}{\partial\theta}\right) - \frac{\partial^2\epsilon_r}{\sin^2\beta\partial\theta^2} + \\
&\qquad \frac{\partial}{\partial r}\left(r^2\frac{\partial\epsilon_\theta}{\partial r}\right) - r\frac{\partial\epsilon_r}{\partial r} \\
&\text{where} \\
\bar{\nabla}^4 &= \bar{\nabla}^2\bar{\nabla}^2 \\
\bar{\nabla}^2 &= (\partial^2/\partial r^2) + (1/r)(\partial/\partial r) + (1/r^2\sin^2\beta)(\partial^2/\partial\theta^2)
\end{aligned}\right\} \tag{3}
$$

For convenience, the following detonations are introduced:

$$
\left.\begin{aligned}
\chi &= \log_e(r/r_1) \\
r_1 &= \text{distance along the generator from the vertex to a point at the smaller end} \\
\varphi &= \theta\sin\beta \qquad F = \bar{F}/Eh^2 \qquad w = \bar{w}/h \\
\nabla^2 &= r_1^2e^{2\chi}\bar{\nabla}^2 = (\partial^2/\partial\chi^2) + (\partial^2/\partial\varphi^2) \\
\nabla^4 &= r_1^4e^{4\chi}\bar{\nabla}^4 = (\partial^4/\partial\chi^4) - 4(\partial^3/\partial\chi\partial\varphi^2) - \\
&\quad 4(\partial^3/\partial\chi^3) + 4(\partial^2/\partial\chi^2) + 2(\partial^4/\partial\chi^2\partial\varphi^2) + \\
&\quad (\partial^4/\partial\varphi^4) + 4(\partial^2/\partial\varphi^2)
\end{aligned}\right\} \tag{4}
$$

After the relations in Eqs. (1) and (4) are substituted into into (3), the dimensionless compatibility equation has the following form:

$$
\nabla^4F = -\left(\frac{r_1}{h}\cot\beta\right)e^{\chi}\left(\frac{\partial^2w}{\partial\chi^2} - \frac{\partial w}{\partial\chi}\right) - \frac{r_1^2}{h^2}e^{2\chi}\nabla^2(\alpha T) \tag{5}
$$

In Eq. (5),

$$
T = \frac{1}{h}\int_{-h/2}^{h/2}\bar{T}(z,t)dZ \tag{6}
$$

and z is the distance from a point in the shell to the middle surface.

The dimensionless equilibrium equation in the normal direction is expressed as

$$
\bar{D}e^{-4\chi}\nabla^4w - \left(\frac{r_1}{h}\cot\beta\right)e^{-3\chi}\left(\frac{\partial^2F}{\partial\chi^2} - \frac{\partial F}{\partial\chi}\right) + \rho\frac{\partial^2w}{\partial t^2} + e^{-2\chi}\nabla^2M_T = 0 \tag{7}
$$

where

$$
\bar{D} = 1/12(1-\nu^2) \tag{8}
$$

Also first to appear in Eq. (7) are the following dimensionless terms:

$$
\rho = (\bar{\rho}r_1^4/Eh^2)(t^2/\bar{t}^2) \tag{9}
$$

$$
M_T = \frac{r_1^2}{(1-\nu)h^4}\int_{-h/2}^{h/2}\alpha\bar{T}ZdZ \tag{10}
$$

In the previous equations, $\bar{t}$, $\bar{\rho}$, and $\bar{T}$ are the dimensional time, mass density, and temperature distribution, respectively. The term $(\bar{t}/t)$ is taken as one unit of time, for instance: $\bar{t}/t = 1$ sec. Equations (5) and (7) are the basic equations for the thermally induced vibrations of conical shells.

Axisymmetric Solution

In this investigation the temperature field varies only in the transverse direction. The vibration mode is taken to be axisymmetric. The stress function F and deflection function w to be solved thus are independent of φ. From the definitions in Eqs. (4) and Fig. 1, the value of χ at the larger end is χ_0, and $\chi = 0$ at the smaller end. At both ends, the cones are simply supported and have zero circumferential strain, but they are free from restrictions on thermal expansion. The conditions at the ends $\chi = 0$ and $\chi = \chi_0$ are accordingly expressed as:

$$
w = 0 \tag{11}
$$

$$
(\partial^2w/\partial\chi^2) - [1-\nu](\partial w/\partial\chi) + (e^{2\chi}/\bar{D})M_T = 0 \tag{12}
$$

$$
[(\partial^2/\partial\chi^2) - (1+\nu)(\partial/\partial\chi)]F = 0 \tag{13}
$$

The solution is made by assuming

$$
w = w_1 + w_2 \tag{14}
$$

such that

$$
\bar{D}\nabla^4w_1 + e^{2\chi}\nabla^2M_T = 0 \tag{15}
$$

and

$$
\bar{D}e^{-4\chi}\nabla^4w_2 + \rho\frac{\partial^2w_2}{\partial t^2} + \rho\frac{\partial^2w_1}{\partial t^2} - \left(\frac{r_1}{h}\cot\beta\right)e^{-3\chi}\left(\frac{\partial^2F}{\partial\chi^2} - \frac{\partial F}{\partial\chi}\right) = 0 \tag{16}
$$

At the ends $\chi = 0$ and $\chi = \chi_0$, the conditions are

$$
\left.\begin{aligned}
&w_1 = w_2 = 0 \\
&(\partial^2w_1/\partial^2\chi^2) - (1-\nu)(\partial w_1/\partial\chi) + (e^{2\chi}/\bar{D})M_T = 0 \\
&(\partial^2w_2/\partial\chi^2) - (1-\nu)(\partial w_2/\partial\chi) = 0
\end{aligned}\right\} \tag{17}
$$

Similarly, the stress function F is expressed as

$$
F = F_1 + F_2 + F_3 + F_T \tag{18}
$$

and

$$\left.\begin{aligned}\nabla^4 F_1 &= -[(r_1/h)\cot\beta]\, e^{\chi}\, [(\partial^2 w_1/\partial\chi^2) - (\partial w_1/\partial\chi)] \\ \nabla^4 F_2 &= -[(r_1/h)]\cot\beta\, e^{\chi}\, [(\partial^2 w_2/\partial\chi^2) - (\partial w_2/\partial\chi)] \\ \nabla^4 F_3 &= 0 \\ \nabla^4 F_T &= -(r_1/h)^2 e^{2\chi}\, \nabla^2(\alpha T)\end{aligned}\right\} \quad (19)$$

The boundary condition for F is found in Eq. (13). Since the temperature distribution is uniform in the middle plane and the ends are free from thermal restriction in the present case, $F_T = 0$. The solution to Eq. (15) is

$$w_1 = -(M_T/4\bar{D})(A_0 + A_1\chi + A_2 e^{2\chi} + A_3 e^{2\chi}\chi) \quad (20)$$

The expressions of A_0, A_1, and A_3 are determined from the conditions in Eq. (17). Upon substituting w_1 of Eq. (20) into the first equation of (19) gives

$$F_1 = -\left(\frac{r_1}{h}\cot\beta\right)\left(\frac{M_T}{4\bar{D}}\right) \times \left[\frac{2A_0 + \frac{7}{3}A_3}{9}\, e^{3\chi} + A_1 e^{\chi} - \frac{2}{9}A_3\chi e^{3\chi}\right] \quad (21)$$

The expression of w_2 has the form

$$w_2 = \sum_{m=1}^{N} Q_m(t) e^{\lambda\chi} \sin\frac{m\pi}{\chi_0}\chi \quad (22)$$

where $Q_m(t)$ is a dimensionless function of time. The factor $e^{\lambda\chi}$ is used to satisfy the simply supported end condition when $\lambda = (1 - \nu)/2$ (Ref. 13). In the following, n is introduced such that

$$n = m\pi/\chi_0 \quad (23)$$

From Eq. (22) and the second equation of (19),

$$F_2 = -\left(\frac{r_1}{h}\cot\beta\right) e^{(1+\lambda)\chi} \sum_{m=1}^{N} Q_m(t)(f_3 \sin n\chi + f_4 \cos n\chi) \quad (24)$$

The function F_3 assumes the form

$$F_3 = -[(r_1/h)\cot\beta](B_1\chi + B_2 e^{2\chi}) \quad (25)$$

The coefficients B_1 and B_2 to be determined from the condition in Eq. (13) are functions of $Q_m(t)$ and $M_T(t)$.

After substitution of the relations in Eqs. (20–25) into Eq. (16), the Galerkin method is applied to establish a set of second-order differential equations. This method involves multiplying the left-hand side of Eq. (16) by $e^{(2+\lambda)\chi} \sin(n\chi)d\chi$ and integrating the product between the limits $\chi = 0$ and $\chi = \chi_0$. In the present study, a solution is obtained from the use of one term, i.e., $m = N$, and $n = N\pi/\chi_0$. The final differential equation has the form

$$\frac{d^2 Q_N}{dt^2} + \omega^2 Q_N = D_i \frac{d^2 M_T}{dt^2} + \left(\frac{r_1}{h}\cot\beta\right)^2 D_2 M_T \quad (26)$$

The notation ω represents the natural frequency, which is

$$\omega^2 = (K_1 n/\rho)\{K_2 - [(r_1/h)\cot\beta]^2 K_3\} \quad (27)$$

The expressions K_1, K_2, K_3, D_1, and D_2 are functions of ν, n, and x_0 but are independent of r_1/h and β. Their relations are listed in the Appendix.

From Eq. (26), the quasi-static solution for Q_N is

$$(Q_N)_{St} = [(r_1/h)\cot\beta]^2 (D_2/\omega^2) M_T \quad (28)$$

The complete solution to Eq. (26) is

$$Q_N = a_1 \cos\omega t + a_2 \sin\omega t + Q_p \quad (29)$$

where Q_p is a particular solution that depends on M_T and its second derivative. The constants a_1 and a_2 are to be determined from the initial conditions specified for w, which is the sum of w_1 and w_2.

Thermal Moment and Initial Conditions

To afford comparison with the cases solved previously on beams[10] and plates,[11] the same heat-conduction relation is adopted for the numerical calculation to be followed. The shell is suddenly heated by a constant heat-input at one side and insulated at the other. From Eq. (10) in this paper,

$$M_T = \frac{2r_1^2\alpha q}{\lambda\pi^4 hK}\left[\frac{\pi^4}{96} - \sum_{j=1,3}^{\infty} \frac{1}{j^4} \exp\left(-j^2\pi^2 \frac{A\bar{t}}{h^2}\right)\right] \quad (30)$$

where q, K, and A are heat input, thermal conductivity, and thermal diffusivity, respectively. When Eq. (30) is used in Eq. (26), the particular solution Q_p in Eq. (29) is found such that

$$Q_p(t) = \frac{r_1^2\alpha q}{48\lambda hK}\left\{\left(\frac{r_1}{h}\cot\beta\right)^2 \frac{D_2}{\omega^2}(1 - \cos\omega t) - 96\sum_{j=1,3}^{\infty}\left[\left(\frac{At}{h^2\bar{t}}\right)^2 D_1 + \frac{D_2}{j^4\pi^4}\left(\frac{r_1}{h}\cot\beta\right)^2\right]\tau_j\right\} \quad (31)$$

where

$$\tau_j = \frac{\{\exp[-j^2\pi^2(A\bar{t}/h^2)] + (j^2\pi^2 A\bar{t}/h^2\omega t)\sin\omega t - \cos\omega t\}}{[(j^4\pi^4 A^2\bar{t}^2/h^4 t^2) + \omega^2]} \quad (32)$$

The initial conditions are assumed,

$$(w)_{t=0} = (\partial w/\partial t)_{t=0} = 0 \quad (33)$$

The first condition finds $a_1 = 0$. The second is satisfied by the variational method.[14] This yields

$$a_2 = -\frac{\int_0^{\chi_0}\left[\left(\frac{\partial w_1}{\partial t}\right)_{t=0}\right]\exp[(2+\lambda)\chi]\sin n\chi\, d\chi}{\omega\int_0^{\chi_0}\exp[2(1+\lambda)\chi]\sin^2 n\chi\, d\chi} \quad (34)$$

The deflection function is now obtained as below,

$$w = \left[Q_p + \frac{r_1^2\alpha q}{48\lambda hK}\left(\frac{96 D_1 A}{\pi^2 h^2\bar{\omega}}\sum_{j=1,3}^{\infty}\frac{1}{j^2}\right)\sin\omega t\right] e^{\lambda\chi}\sin n\chi - \frac{M_T}{4\bar{D}}(A_0 + A_1\chi + A_2 e^{2\chi} + A_3\chi e^{2\chi}) \quad (35)$$

where Q_p and M_T are expressed in Eqs. (31) and (30). The functions A_i and D_j are listed in the Appendix. The deflection function from quasi-static consideration is

$$w_{St} = M_T[[(r_1/h)\cot\beta]^2 (D_2/\omega^2) e^{\lambda\chi}\sin n\chi - 1/4\bar{D}(A_0 + A_1\chi + A_2 e^{2\chi} + A_3\chi e^{2\chi})] \quad (36)$$

The dynamic effects can be shown by comparing the magnitudes of deflections found from Eqs. (35) and (36). The deflection ratio is defined as the ratio between the maximum w and the maximum static deflection, or it is expressed as

$$\text{deflection ratio} = w_{\max}/(w_{St})_{\max} \quad (37)$$

At any time, the deflection ratio is independent of $\alpha q/K$, which is a common factor in the expressions w and w_{St}.

Numerical Example and Discussions

Some numerical calculations have been made for studying the free vibration frequencies and the deflection ratios. The geometric parameters of conical shells are: the length index χ_0, the semivertex angle β, and the radius thickness ratio

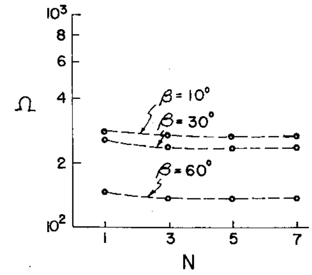

Fig. 2 Frequency parameter vs mode number at various semivertex angles (x_0 = 1.0, R/h = 500).

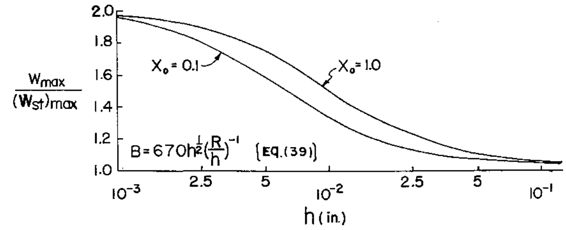

Fig. 5 Variation of deflection ratio with wall thickness (β = 30°, R/h = 500).

R/h (Fig. 1). The conical shells considered for the numerical evaluation are made of aluminum of which $\bar{\rho} = 2.527 \times 10^{-4}$ lb-sec²/in.⁴, Young's modulus $E = 10^1$ psi, and thermal diffusivity $A = 0.1333$ in.²/sec. The frequency equation (27) is used to find the relations shown in Figs. 2 and 3. The frequency parameter Ω stands for

$$\Omega = \bar{\omega}h(R/h)^2(\bar{\rho}/E)^{1/2} = [\bar{\omega}\bar{D}^{1/2}(h^2/A)]/B^2 \qquad (38)$$

where B is a time-ratio parameter first defined by Boley.[10] For cylindrical or conical shells,

$$B = h^{1/2}(R/h)^{-1}(\bar{D}E/A^2\bar{\rho})^{1/4} \qquad (39)$$

It can be seen that, for the same β, the frequency parameter for the shorter cone is higher. When the length-index x_0

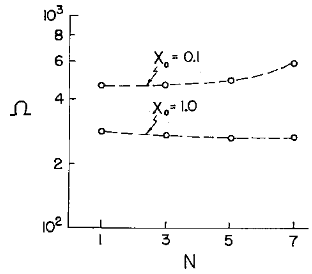

Fig. 3 Frequency parameter vs mode number at different length (semivertex angle = 10°, R/h = 500).

is the same, the frequency parameter changes significantly with respect to the vertex angle.

The deflection ratio has been evaluated at $m = N = 1$ and $\chi = \chi_0/2$. The variations of this ratio with the thickness of shell are plotted in Figs. 4, 5, and 6. The dynamic effect is more apparent for thin shells of high R/h ratio. Similar relationships are expected for the cases of cylindrical shells ($\beta \rightarrow 0$, $\chi_0 \rightarrow 0$) and annular circular plates ($\beta \rightarrow 90°$).

Appendix

In order to write the relations in brief forms, several functions have been represented by symbols. They are defined in the following list:

$$\bar{D} = \tfrac{1}{2}(1 - \nu^2)$$

$$\lambda = (1 - \nu)/2 \qquad \lambda_1 = 1 - \lambda \qquad \lambda_2 = 1 + \lambda$$

$$\lambda_3 = 2 + \lambda \qquad \lambda_4 = 4 + \lambda$$

$$\lambda_5 = 3\lambda^2 + 2\lambda - (1 + n^2)$$

$$b_1 = (1 + 2\lambda)[(6A_0 + 7A_3)/36\bar{D}] - (\lambda_3 A_3/9\bar{D})$$

$$b_2 = -[(1 + 2\lambda)/6\bar{D}]A_3 \qquad b_3 = -[(1 - 2\lambda)/4\bar{D}]A_1$$

$$c_1 = \exp(\chi_0) \qquad c_2 = \exp[(\lambda_1 - 1)\chi_0]$$

$$c_3 = 2\lambda(c_2^2 - 1)$$

$$c_4 = c_3 + 4\lambda_1\chi_0 \qquad c_5 = \frac{c_2^2\chi_0}{c_2^2 - 1} + \frac{1 + \lambda_1}{2\lambda_1}$$

$$c_6 = 2\lambda_1(c_2 - 1)$$

$$d_0 = (1 + c_1^{2+\lambda})/(n^2 + \lambda_3^2)$$

$$d_2 = (1 + c_1^{4+\lambda})/(n^2 + \lambda_4^2)$$

$$d_1 = \frac{c_1^{2+\lambda}[(n^2 + \lambda_3^2)\chi_0 - 2\lambda_3] - 2\lambda_3}{(n^2 + \lambda_3^2)^2}$$

$$d_3 = \frac{c_1^{4+\lambda}[(n^2 + \lambda_4^2)\chi_0 - 2\lambda_4] - 2\lambda_4}{(n^2 + \lambda_4^2)^2}$$

$$d_4 = \frac{b_1(1 - c_1^3) - b_2c_1^3\chi_0 + b_3(1 - c_1)}{2c_3}$$

$$d_5 = \frac{b_1c_1^2(1 - c_1) - b_2c_1^3\chi_0 - b_3c_1(1 - c_1)}{c_6}$$

$$f_1 = \lambda_1\lambda_2(\lambda_1\lambda_2 + 6n^2) + (n^2 - 4)n^2$$

$$f_2 = 4n\lambda(\lambda_1\lambda_2 + n^2)$$

$$f_3 = -\frac{f_1}{f_1^2 + f_2^2}(\lambda_1\lambda + n^2) + \frac{f_2}{f_1^2 + f_2^2}(1 - 2\lambda)n$$

$$f_4 = -\frac{f_2}{f_1^2 + f_2^2}(\lambda_1\lambda + n^2) + \frac{f_1}{f_1^2 + f_2^2}(2\lambda - 1)n$$

$$k_1 = [\bar{D}(c_1^{2+2\lambda} + 1)/4\lambda_1c_1^{2+2\lambda}(n^2 + \lambda_1^2)]$$

$$k_2 = n^4 - 2(3\lambda^2 - 6\lambda + 2)n^2 + \lambda^2(2 - \lambda)^2$$

$$k_3 = 4n^{2\lambda} + 4\lambda(2 - \lambda) \qquad k_4 = (c_1^{2\lambda} - 1)/4\lambda$$

$$k_5 = \frac{(1 + c_1^{1+\lambda})^2}{C_3(n^2 + \lambda_2^2)} - \frac{2c_1^{1+\lambda} + c_1^2 + c_1^{2\lambda}}{C_6(n^2 + \lambda_1^2)}$$

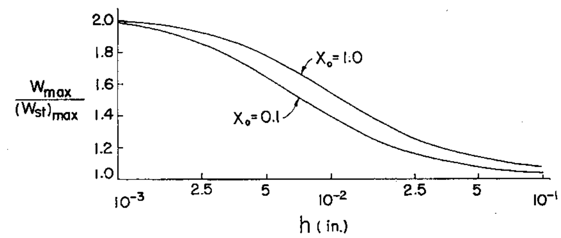

Fig. 4 Variation of deflection ratio with wall thickness (β = 10°, R/h = 500).

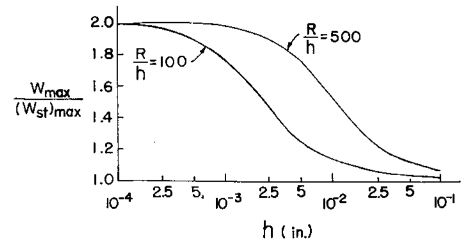

Fig. 6 Variation of deflection ratio at different values of R/h (x_0 = 1.0, β = 30°).

$$A_0 = -A_2 = [-4 + (4 + c_3)A_3]\chi_0/c_4$$

$$A_1 = -4\lambda_1 c_1{}^2 A_3 \chi_0/c_4$$

$$A_3 = c_3[(c_4 c_5 \lambda_1) - (4 + c_3)\lambda_1\chi_0]^{-1}$$

$$D_1 = K_1 n(d_0 A_0 + d_1 A_1 + d_2 A_2 + d_3 A_3)/4\bar{D}$$

$$D_2 = (K_1 n/\rho)[-d_0(3A_0 + A_3)/9\bar{D}] + d_1(A_3/3\bar{D}) - 2d_4(1 + c_1{}^{1+\lambda})/(n^2 + \lambda_2{}^2) + d_5(1 + c_1{}^{1-\lambda})/(n^2 + \lambda_1{}^2)]$$

$$K_1 = \frac{4\lambda_2(n^2 + \lambda_2{}^2)}{n^2(c_1{}^{2+2\lambda} - 1)}$$

$$K_2 = k_1(k_2 n + k_3\lambda_1{}^2)$$

$$K_3 = n(k_4 - 4\lambda k_5)f_3 + (k_4\lambda_2 - \lambda_5 k_5)f_4$$

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